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Electromagnetic wave interaction with varying plasma*

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Owing to the variation in the power absorbed in the plasma from a perturbing high frequency alternating electric field, the plasma electron temperature varies and hence there will be perturbations in the plasma electron density and electron-atom elastic collision frequency. When an electromagnetic wave passes through such a plasma there will be an amplitude modulation impressed on the propagating wave. A simple analysis has been made for the amplitude modulation which could be used to interpret the experiments to be conducted. It is found that the modulation decreases with the increase in the plasma field which depends on the electron-atom collision frequency.

1. INTRODUCTION

When an electromagnetic wave passes through a plasma, owing to the variations in the plasma parameters such as electron density (n), electron-neutral atom elastic collision frequency (ν), electron temperature (T_e) etc. there will be a modulation impressed on the propagating wave in its amplitude as well as in its phase. Ginzburg & Gurovich (1960), Whitmer & Barret (1963), Sodha & Arorra (1969) and Goldstein (1953) made a detailed theoretical study and presented the solutions. However, experimental studies on this phenomena are inadequate. A modest attempt has been made to conduct experiments in this laboratory by John & Sarkar (1970), Chandra *et al* (1970) and Kumar *et al* (1971) and the results already have been published. Here an improved simple theoretical model other than the one already presented by John *et al* has been derived that could be applied to the laboratory conditions. In these experiments, plasma is excited by a modulated radiofrequency (RF) electric power which is expected to produce a periodic variation only in n and ν of the plasma parameters and induce a phase difference between the variable parameters and the exciting field.

A preliminary theoretical analysis is presented which could give explicit equations for α and β of the plasma (perturbed by a high frequency alternating field) where $\alpha = n'/n_0$, n_0 is the equilibrium or direct component of the electron density and n' is the maximum variation in n from the equilibrium value and $\beta = \nu'/\nu_0$ the respective maximum and equilibrium values in ν. α and β have

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been calculated using the equation for the energy absorbed in the plasma from the perturbing field. The impressed amplitude modulation (μ) on the propagating wave has been derived in terms of α and β. The phase difference between varying n and ν with the perturbing field has been neglected for this approximate analysis. The guiding equations for ascertaining the small perturbations in n, ν and T_e of the plasma and μ on the propagating wave are taken as the force equation, the continuity equation and Maxwell's equations. The effect of the plasma field (which in turn depends on ν_0) on the propagating wave has been introduced in evaluating the impressed modulation.

2. Analysis

The plasma is assumed to be a Lorentz gas and initially is at rest. Considered variations are in the associated electric and magnetic fields, electron velocity, number density and elastic collision frequency Let the electric field $\boldsymbol{E} = 0+\boldsymbol{E}'$, the electron velocity $\boldsymbol{v} = 0+\boldsymbol{v}'$, density $n = n_0+n'$ adnd collision frequency $\nu = \nu_0+\nu'$. The terms with subscript zero are the equilibrium or d.c. values while those with primes are the maximum perturbations as a function of space and time.

If the perturbations are kept only to the first order, then the force and continuity equations may be written as,

$$\frac{\partial \boldsymbol{v}'}{\partial t}+\frac{e\boldsymbol{E}'}{m}+\frac{C^2{}_s}{n_0}\left\{\text{grad } n'-\frac{n'}{n_0}\text{ grad } n_0\right\}+\nu\boldsymbol{v}' = 0 \qquad \ldots (1)$$

$$\frac{\partial n'}{\partial t}+n_0 \text{ div } \boldsymbol{v}'+(\boldsymbol{v}'.\text{ grad } n_0) = 0. \qquad \ldots (2)$$

The Maxwell's equations are,

$$\left.\begin{aligned}\frac{\partial \boldsymbol{E}'}{\partial t} &= -\left(\frac{4\pi}{C}\right)\boldsymbol{\tau}' \\ \text{div } \boldsymbol{E}' &= -4\pi q'\end{aligned}\right\} \qquad \ldots (3)$$

where $\boldsymbol{\tau}' = -en_0\boldsymbol{v}'$ and $q' = -en'$.

$C_s = \left(\frac{k.T_e}{m}\right)^{\frac{1}{2}}$ is the thermal velocity of the electrons. k is the Boltzmann constant and C is the velocity of light.

The variation in the number density in space is given by

$$|\text{grad } n_0| \approx \frac{n_0}{h} \qquad \ldots (4)$$

where h is the characteristic length of the variation in equilibrium concentration. Omitting the primes, the variable quantities can be expressed in the form of plane waves as

$$\left.\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_0 \exp i(\boldsymbol{K}.\boldsymbol{r}-\Omega t) \\ \boldsymbol{v} &= \boldsymbol{v}_{00} \exp i(\boldsymbol{K}.\boldsymbol{r}-\Omega t) \\ n &= n_{00} \exp i(\boldsymbol{K}.\boldsymbol{r}-\Omega t) \\ \nu &= \nu_{00} \exp i(\boldsymbol{K}.\boldsymbol{r}-\Omega t) \end{aligned}\right\} \quad \ldots \quad (5)$$

where $\boldsymbol{E}_0$, $\boldsymbol{v}_{00}$, n_{00} and ν_{00} are the amplitudes that are constant in time and space. $\boldsymbol{K}$ is the wave vector and Ω is the frequency of the perturbing field.

From the above equations, it can be shown that

$$i\omega'\boldsymbol{v} - \frac{e}{m}\boldsymbol{E}_1 = 0 \quad \ldots \quad (6)$$

$$n + \frac{in_0}{\Omega}\left(\text{div}\,\boldsymbol{v} + \frac{|\boldsymbol{v}|}{h}\right) = 0 \quad \ldots \quad (7)$$

$$\text{div}\,\boldsymbol{E} + 4\pi en = 0 \quad \ldots \quad (8)$$

where $\omega' = \Omega + i\nu$; and

$$\boldsymbol{E}_1 = \boldsymbol{E} + \frac{mC_s^2}{e}\left(\text{grad}\,\frac{n}{n_0} - \frac{n}{hn_0}\right) \quad \ldots \quad (9)$$

Evaluation of the above equations gives the following solution for α

$$\alpha = \frac{n}{n_0} = \frac{\epsilon E_0}{mh}\{(\omega_p^2 - K^2C_s^2 - \Omega^2)^2 + \nu^2\Omega^2\}^{\frac{1}{2}}. \quad \ldots \quad (10)$$

where the plasma frequency $\omega_p = \left(\frac{ne^2}{m\epsilon}\right)^{\frac{1}{2}}$; and ϵ being the dielectric permitivity of the medium.

Owing to the variation in the power absorbed in the plasma from the perturbing field, the electron temperature varies and hence changes the electron collision frequency. The energy balance for such a plasma would be

$$\frac{dT_\epsilon}{T_e} = \left(\frac{2e}{3k}\right)\boldsymbol{v}.\boldsymbol{E} - \delta\nu_{T_e}(T_\epsilon - T) \quad \ldots \quad (11)$$

where $\delta = 2m/M$ the fraction of energy lost in an elastic collision and T is the gas temperature.

Following Ginzburg & Gurevich (1960), the expression for the temperature perturbation can be written as, with $\Delta T_e = (T_e - T)$

$$\frac{\Delta T_e}{T} = \frac{\alpha e^2 E_0^2}{3mkT\delta(\nu_0^2+\Omega^2)} \left\{ \frac{\delta\nu_0 \cos(\Omega t+\phi_\Omega)}{[(\delta\nu_0)^2+\Omega^2]^{\frac{1}{2}}} + \frac{\alpha\delta\nu_0 \cos(2\Omega t-\phi_{2\Omega})}{[(\delta\nu_0)^2+(2\Omega)^2]^{\frac{1}{2}}} \right\} \quad \ldots (12)$$

where

$$\phi_\Omega = \tan^{-1}\left(\frac{\Omega}{\delta\nu_0}\right) \text{ and } \phi_{2\Omega} = \tan^{-1}\left(\frac{2\Omega}{\delta\nu_0}\right)$$

Since

$$\nu = \nu_0 + \nu' \quad \text{and} \quad \nu = \nu_0\sqrt{\frac{T_e}{T}}$$

$$\nu = \nu_0\left(1+\frac{\Delta T_e}{T}\right)^{\frac{1}{2}}.$$

Therefore

$$\nu' = \frac{\nu_0 \Delta T_e}{2T} \quad \ldots (13)$$

Neglecting higher order terms,

$$\frac{\nu'}{\nu_0} = \beta = \frac{\alpha}{2} \cdot \left[\frac{e^2E_0^2}{3mkT\delta(\nu_0^2+\Omega^2)\left[1+\left(\frac{\Omega}{\delta\nu_0}\right)^2\right]^{\frac{1}{2}}}\right] \quad \ldots (14)$$

The characteristic plasma field E_p can be defined as

$$E_p = \left[\frac{3mkT\delta}{e^2}(\nu_0^2+\Omega^2)\right]^{\frac{1}{2}} \quad \ldots (15)$$

Thus eq. (14) in terms of eq. (15) can be written as,

$$\beta = \frac{\alpha}{2}\left(\frac{E_0}{E_p}\right)^2 \frac{1}{\left[1+\left(\frac{\Omega}{\delta\nu_0}\right)^2\right]^{\frac{1}{2}}} \quad \ldots (16)$$

With the periodic variation in n and ν as given by eqs. (10) and (16), solving Maxwell's equations in terms of the fluctuating plasma conductivity, the modulation impressed on the propagating wave can be derived (John & Sarkrar, 1970),

$$\mu = \frac{\pi\left(\frac{\omega_p^2}{\omega^2}\right)\left(\frac{Z}{\lambda}\right)}{\left\{1+\left(\frac{\nu_0}{\omega}\right)^2\right\}^{\frac{1}{2}}}\left[\left\{\alpha-\frac{\beta}{1+\left(\frac{\omega}{\nu_0}\right)^2}\right\}^2+\left\{\frac{\beta\left(\frac{\nu_0}{\omega}\right)}{1+\left(\frac{\nu_0}{\omega}\right)^2}\right\}^2\right]^{\frac{1}{2}} \quad (17)$$

where Z is the propagation length, ω is the frequency and λ is the free space wavelength of the propagating electromagnetic wave.

3. Conclusions

The variation in electron density and elastic collision frequency characterized by α and β are evaluated from the explicit expressions given by eqs. (10) and (16) respectively. The amplitude modulation impressed on the propagating wave has been calculated from eq. (17). Figure 1 represents the dependence of μ on

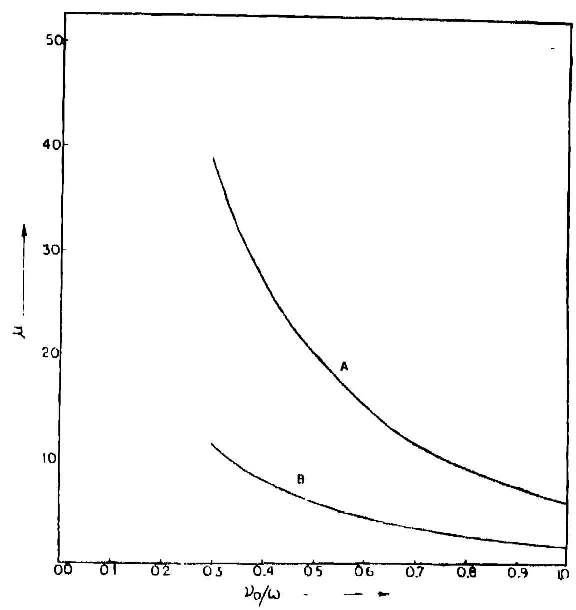

Fig 1. Variation of the impressed amplitude modulation percentage (μ) with ω_p^2/ω^2 (Curve A) and with perturbing field E_0 (Curve B).

$\omega p^2/\omega^2$ and E_0 (keeping ν_0/ω constant) It is evident that μ increases almost linearly with $\omega p^2/\omega^2$ in accordance with Sodha & Arora (1969) and μ increases steadily with E_0 as shown by Ginzburg & Gurevich (1960). Figure 2 shows the variation of μ with ν_0/ω, the collision frequency parameter. It is interesting to note that with the increase in ν_0 which increases the plasma field, there is first a sharp decrease in μ and then a slow fall. This fact also has been experimentally observed by Varshney *et al* (1976).

In conclusion it can be said that an electromagnetic wave passing through a time dependent plasma, gets strongly modulated in its amplitude. The impressed modulation decreases with the increase in the plasma field or with the increase in the electron-atom elastic collision frequency.

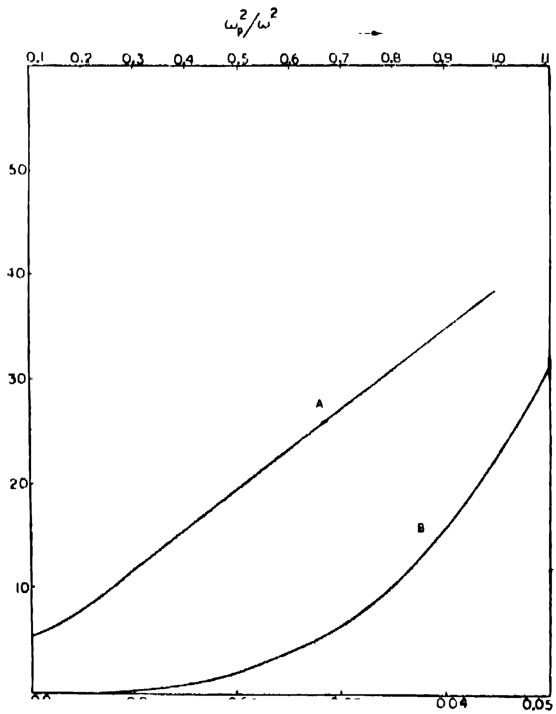

Fig. 2. Variation of μ with ν_0/ω for $\omega_p{}^2/\omega^2 = 1$ (Curve A) and $\omega_p{}^2/\omega^2 = 0.3$ (Curve B).

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